Analysis and Implementation of a High-Order Reconstruction Algorithm for an Unstructured Finite Volume Flow Solver

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Outline

High-Order Unstructured Finite Volume Methods

- Structured Methods:
  - ENO - Harten, Enquist, Osher, and Chakravarthy.
  - WENO - Liu, Osher, and Chan.
- Barth and Frederickson: Seminal Paper for Higher Order on Unstructured Grids.
- ENO Ideas Introduced by Harten and Chakravarthy and Abgrall.
- Ollivier-Gooch: Examined Method.

Research Goals

- High-Order Solutions for Equations of Fluid Dynamics.
- Extendable to Tenasi:
  - Parallelizable
  - Support for Cell/Vertex-Centered Formulation
  - Element Neutral

Flow Solver

Solves the 2D Euler Equations

- Vertex-Centered, Median Dual
- Roe Scheme
- CVBCs
- Spatial Accuracies - 1^{st} through 4^{th}
- Temporal Accuracies:
  - Explicit - 1^{st}-Order Forward Euler
  - Implicit - 1^{st}-Order Backward Euler, 2^{nd}-Order Finite Diff. App.
- Approximate Flux Linearization
- Symmetric Gauss-Seidel Linear Solver

Three Criteria for High-Order Reconstruction (from Barth and Frederickson)

Reconstruction Algorithm

Three Criteria for High-Order Reconstruction (from Barth and Frederickson)

- Conservation in the Mean

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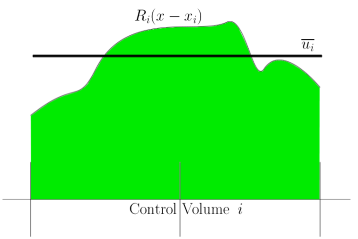
Three Criteria for High-Order Reconstruction (from Barth and Frederickson)

- Conservation in the Mean



Three Criteria for High-Order Reconstruction (from Barth and Frederickson)

- Conservation in the Mean
- *k*-Exact Reconstruction

Reconstruction Algorithm

Three Criteria for High-Order Reconstruction (from Barth and Frederickson)

- Conservation in the Mean
- k-Exact Reconstruction
- Compact Support

Final Form:

$$\frac{1}{V_j}\int_{V_j} R_i(\vec{x}-\vec{x_i})dV = u\Big|_{\vec{x_i}} + \frac{\partial u}{\partial x}\Big|_{\vec{x_i}}\left(\overline{x_j}+(x_j-x_i)\right)$$
$$+\frac{\partial u}{\partial y}\Big|_{\vec{x_i}}\left(\overline{y_j}+(y_j-y_i)\right)+\frac{\partial^2 u}{\partial x^2}\Big|_{\vec{x_i}}\left(\frac{1}{2}(\overline{x_j^2}+2\overline{x_j}(x_j-x_i)+(x_j-x_i)^2)\right)$$
$$+\frac{\partial^2 u}{\partial y^2}\Big|_{\vec{x_i}}\left(\frac{1}{2}(\overline{y_j^2}+2\overline{y_j}(y_j-y_i)+(y_j-y_i)^2)\right)$$
$$+\frac{\partial^2 u}{\partial x\partial y}\Big|_{\vec{x_i}}\left(\overline{xy_j}+\overline{x_j}(y_j-y_i)+\overline{y_j}(x_j-x_i)+(x_j-x_i)(y_j-y_i)\right)$$

Least Squares System:

$$\left[\begin{array}{cccccc} 1 & \overline{x} & \overline{y} & \overline{x^2} & \overline{y^2} & \overline{xy} \\ \hline w_{i1} & w_{i1}\widehat{x_{i1}} & w_{i1}\widehat{y_{i1}} & w_{i1}\widehat{x_{i1}^2} & w_{i1}\widehat{y_{i1}^2} & w_{i1}\widehat{xy_{i1}} \\ w_{i2} & w_{i2}\widehat{x_{i2}} & w_{i2}\widehat{y_{i2}} & w_{i2}\widehat{x_{i2}^2} & w_{i2}\widehat{y_{i2}^2} & w_{i2}\widehat{xy_{i2}} \\ w_{i3} & w_{i3}\widehat{x_{i3}} & w_{i3}\widehat{y_{i3}} & w_{i3}\widehat{x_{i3}^2} & w_{i3}\widehat{y_{i3}^2} & w_{i3}\widehat{xy_{i3}} \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ w_{in} & w_{in}\widehat{x_{in}} & w_{in}\widehat{y_{in}} & w_{in}\widehat{x_{in}^2} & w_{in}\widehat{y_{in}^2} & w_{in}\widehat{xy_{in}} \end{array}\right] \begin{pmatrix} u \\ \frac{\partial u}{\partial x} \\ \frac{\partial u}{\partial y} \\ \frac{1}{2}\frac{\partial^2 u}{\partial x^2} \\ \frac{1}{2}\frac{\partial^2 u}{\partial y^2} \\ \frac{\partial^2 u}{\partial x \partial y} \end{pmatrix}_i = \left(\begin{array}{c} \overline{u_i} \\ \hline w_{i1}\overline{u_1} \\ w_{i2}\overline{u_2} \\ w_{i3}\overline{u_3} \\ \vdots \\ w_{in}\overline{u_n} \end{array}\right)$$

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Reconstruction Algorithm
Implementation, continued

$$\widehat{x^a y_{ij}^b} = \sum_{d=0}^{b} \sum_{c=0}^{a} \left(\frac{b!}{d!(b-d)!} \frac{a!}{c!(a-c)!} (x_j - x_i)^c (y_j - y_i)^d \overline{x^{a-c} y^{b-d}}_j \right)$$

Geometric Weighting Parameter:

$$w_{ij} = \frac{1}{|\vec{x}_j - \vec{x}_i|^p}, \; p \in \{0, 1, 2\}$$

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Three Notes:

1. Mean Constraint is Eliminated
2. $\overline{u_i}$ is Replaced with Actual Flow Variable
3. Reconstruct Either Conserved or Primitive Variables

$$2^{nd}: \quad \vec{Q}_{interface} = \vec{Q}_i + \nabla \vec{Q}_i \cdot \vec{r}$$

$$3^{rd}: \quad \vec{Q}_{interface} = \vec{Q}_i + \nabla \vec{Q}_i \cdot \vec{r} + \frac{1}{2}\vec{r}^T \cdot \nabla^2 \vec{Q}_i \cdot \vec{r}$$

$$4^{th}: \quad \vec{Q}_{interface} = \vec{Q}_i + \nabla \vec{Q}_i \cdot \vec{r} + \frac{1}{2}\vec{r}^T \cdot \nabla^2 \vec{Q}_i \cdot \vec{r} + \frac{1}{6}(\vec{r} \cdot (\vec{r} \cdot (\vec{r} \cdot \nabla^3 \vec{Q}_i)))$$

$$\vec{r} = \vec{x}_{interface} - \vec{x}_i$$

Important Details:

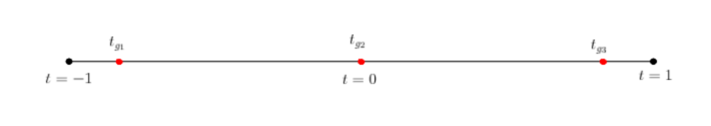
- 2^{nd}-Order - Midpoint Rule, CV average value for $\vec{Q}_i$
- Higher Orders - Need More Accurate Integration, Must Use Node Value in Reconstruction (See Harten and Chakravarthy)

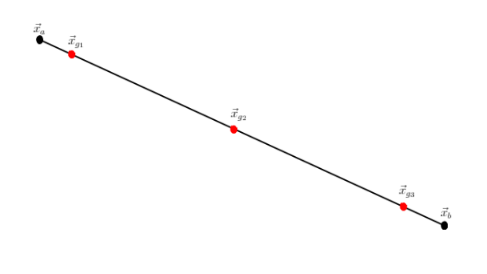
Three Point Gaussian Quadrature

t	Weight
0	$8/9$
$\pm\sqrt{3/5}$	$5/9$

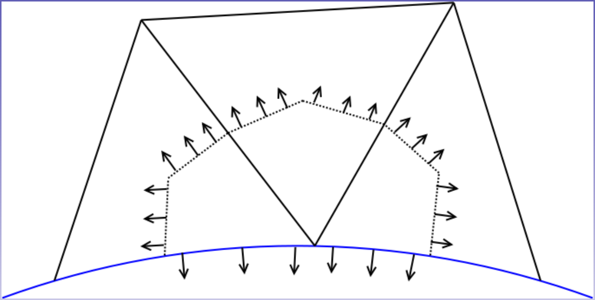
Parameterization - $\vec{x}(t) = \frac{1}{2}\left(\vec{x}_a + \vec{x}_b\right) + \frac{1}{2}\left(\vec{x}_b - \vec{x}_a\right) t$

Quadrature Node Locations

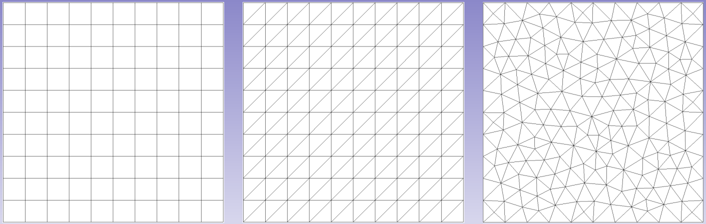




Constant Radius: Based on Angle, $\theta(t) = \frac{1}{2}(\theta_a + \theta_b) + \frac{1}{2}(\theta_b - \theta_a)\, t$



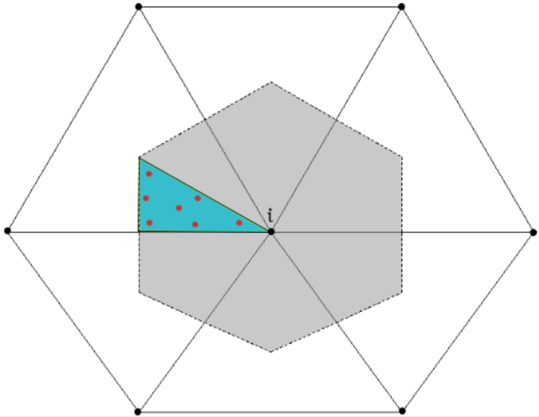
$$f_1(x,y) = x^2 + y^2 + xy + x + y$$
$$f_2(x,y) = 3x^3 + 5xy^2$$
$$f_3(x,y) = sin(\pi x)cos(\pi y)$$
$$f_4(x,y) = e^{-r^2},\ \ r^2 = \left(x - \frac{1}{2}\right)^2 + \left(y - \frac{1}{2}\right)^2$$

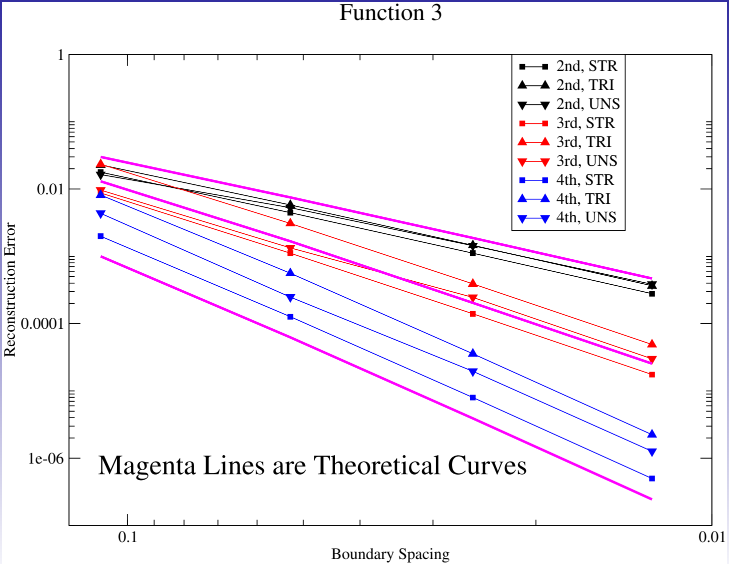


Test Procedure:

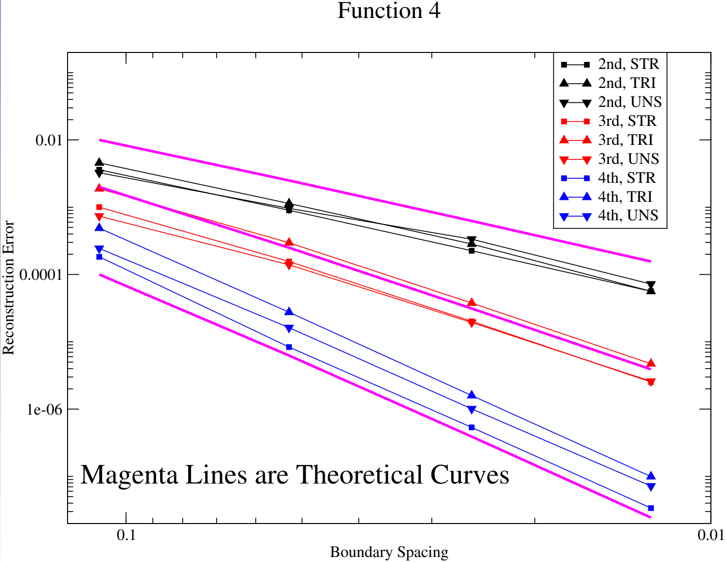
1. Initialize CV Averages - Divergence, Triangle Integration
2. Solve Least Squares - $p = 0$
3. Track Maximum Error Between Exact and Reconstruction

Quadrature Nodes on a Constituent Triangle




Function 3
Reconstruction Error
1
0.01
0.0001
1e-06
0.1
0.01
Boundary Spacing
2nd, STR
2nd, TRI
2nd, UNS
3rd, STR
3rd, TRI
3rd, UNS
4th, STR
4th, TRI
4th, UNS
Magenta Lines are Theoretical Curves

Reconstruction Algorithm
Smooth Function Test, Function 4 Results



Original Reconstruction:

$$u_G = u(\vec{x}_i) + S(\vec{x}_G - \vec{x}_i) + H(\vec{x}_G - \vec{x}_i)$$

With Slope Limiter:

$$u_G = u(\vec{x}_i) + \phi_i(S(\vec{x}_G - \vec{x}_i) + H(\vec{x}_G - \vec{x}_i)), \phi \in [0, 1]$$

Implemented Limiters:

1. Barth and Jespersen
2. Venkatakrishnan
3. Nejat and Ollivier-Gooch
4. Michalak and Ollivier-Gooch

1. Barth and Jespersen

1. Barth and Jespersen
   Find largest admissible ϕ; Strictly monotone

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2. Venkatakrishnan

1. Barth and Jespersen
   Find largest admissible ϕ; Strictly monotone
2. Venkatakrishnan
   Differentiable; Monotonicity not strictly enforced

Reconstruction Algorithm
Solution Monotonicity, continued

1. Barth and Jespersen
   Find largest admissible ϕ; Strictly monotone
2. Venkatakrishnan
   Differentiable; Monotonicity not strictly enforced
3. Nejat and Ollivier-Gooch

Reconstruction Algorithm

Solution Monotonicity, continued

1. Barth and Jespersen
   Find largest admissible ϕ; Strictly monotone
2. Venkatakrishnan
   Differentiable; Monotonicity not strictly enforced
3. Nejat and Ollivier-Gooch
   Previous limiters too diffusive; Add separate limiter for H.O.T.

1. Barth and Jespersen
   Find largest admissible ϕ; Strictly monotone
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   Previous limiters too diffusive; Add separate limiter for H.O.T.
4. Michalak and Ollivier-Gooch
   Venkatakrishnan's min function not sufficient

Add a Source Term:

$$\frac{\partial}{\partial t}\int_{CV}\vec{Q}\,dV + \oint_{CS}\vec{F}\cdot\hat{n}\,d\vec{A} = S(x,y)$$

Flux of the Manufactured Solution:

$$S(x,y) = \oint_{CS_i}\vec{F}(\vec{Q}^E)\cdot\hat{n}\,d\vec{A}_i$$

Modify the Right Hand Side;

$$\left[\frac{V_i}{\triangle t}I + \frac{\partial\Re}{\partial Q}^m\right]\triangle\overline{Q} = -\Re(\overline{Q}^m) + \Re(\vec{Q}^E)$$

Method of Manufactured Solutions
Continued

Exact Solution:

$$\begin{aligned}
\rho &= 1 + \frac{1}{4}\sin(\pi x)\sin(\pi y) \\
u &= \frac{1}{4} + \frac{1}{4}\sin(\pi x)\cos(2\pi y) \\
v &= \frac{1}{4} + \frac{1}{4}\cos(2\pi x)\sin(\pi y) \\
P &= \frac{1}{\gamma} + \frac{1}{20}\cos(2\pi x)\cos(2\pi y)
\end{aligned}$$

Evaluate as Area Integral Rather Than Contour

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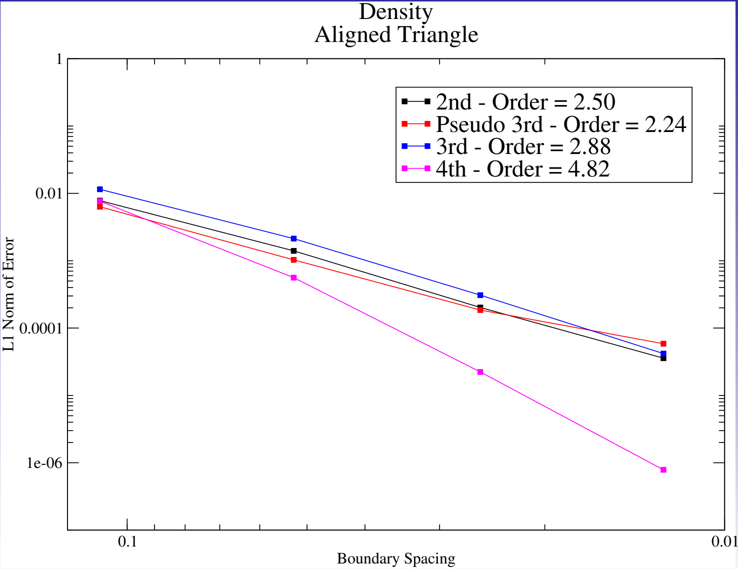
Use Same Grids From Smooth Function Verification Test:

- 2^{nd}-Order
- Pseudo 3^{rd}-Order (Quadratic Extrapolation)
- 3^{rd}-Order
- 4^{th}-Order

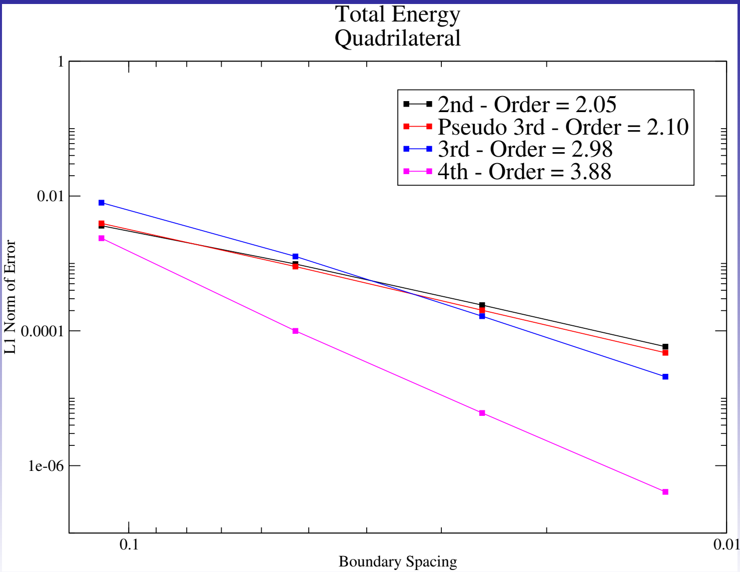
Method of Manufactured Solutions

Linear Boundaries Results, Density Error from Aligned Triangles, L_1

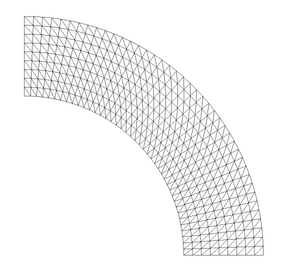


Method of Manufactured Solutions

Linear Boundaries Results, Total Energy Error from Quadrilateral, L_1



Annular Geometry with $r_{inner} = 2$ and $r_{outer} = 3$



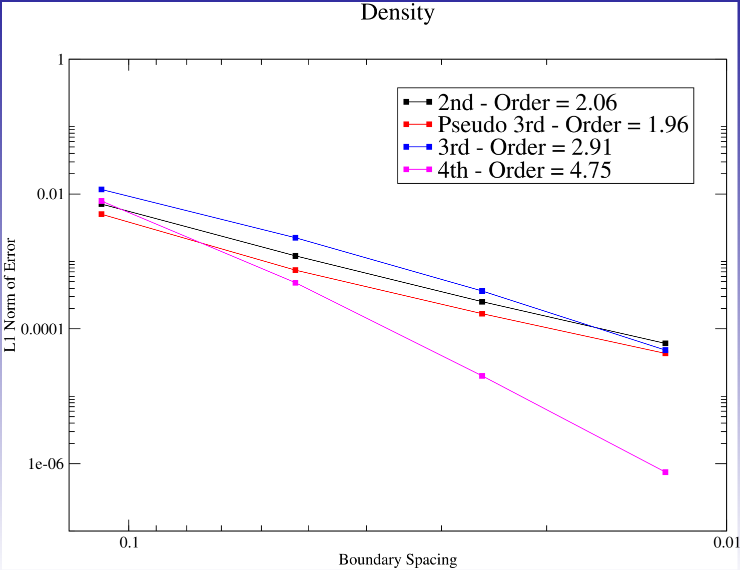
Test Same Methods

Curved Boundary $\Rightarrow$ Triangles with One Curved Side

Evaluate Area Integrals with Isoparametric Mapping (Quadratic)

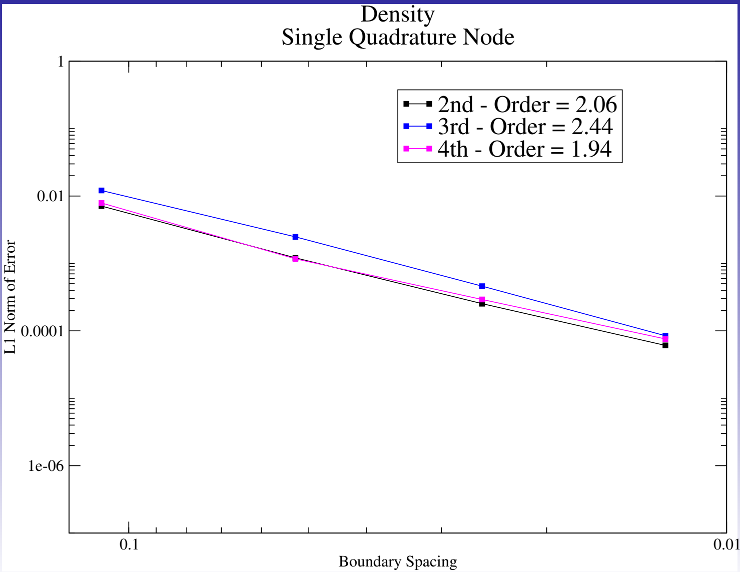
Method of Manufactured Solutions

Curved Boundaries Results, Density Error



Method of Manufactured Solutions

Curved Boundaries Results, Convergence with Midpoint Rule



- Supersonic Flow in an Annulus
- Subsonic Flow Over a Circular Cylinder
- Flow Over the NACA 0012 Airfoil

Analytical Solution: $\rho_i = 1,\ M_i = 2,\ R_i = 2$

$$\rho = \rho_i \left(1 + \frac{\gamma-1}{2} M_i^2 \left(1 - \frac{R_i^2}{r^2}\right)\right)^{\frac{1}{\gamma-1}}$$

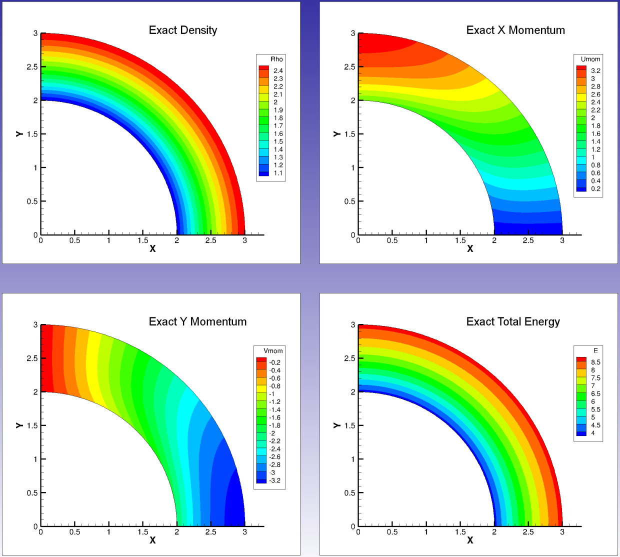
$$U_i = M_i \rho_i^{\frac{\gamma-1}{2}} \qquad U = \frac{U_i R_i}{r}$$

$$u = \frac{yU}{r} \qquad v = \frac{-xU}{r}$$

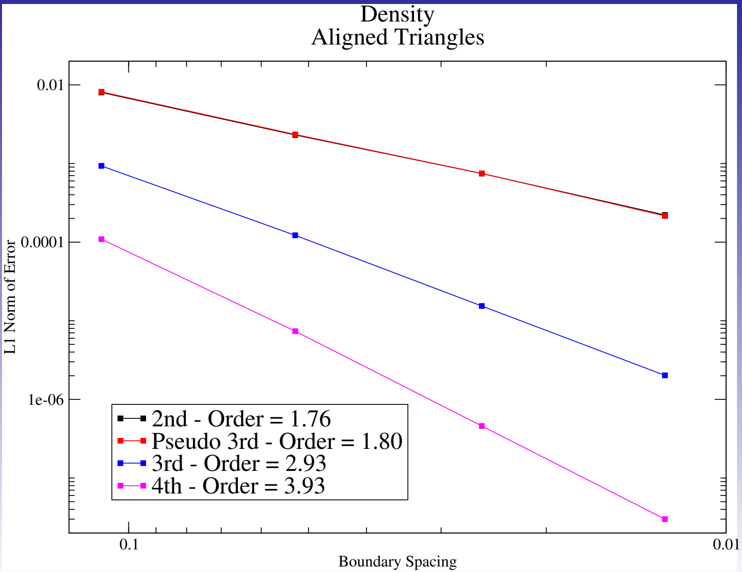
$$P = \frac{\rho^\gamma}{\gamma}$$

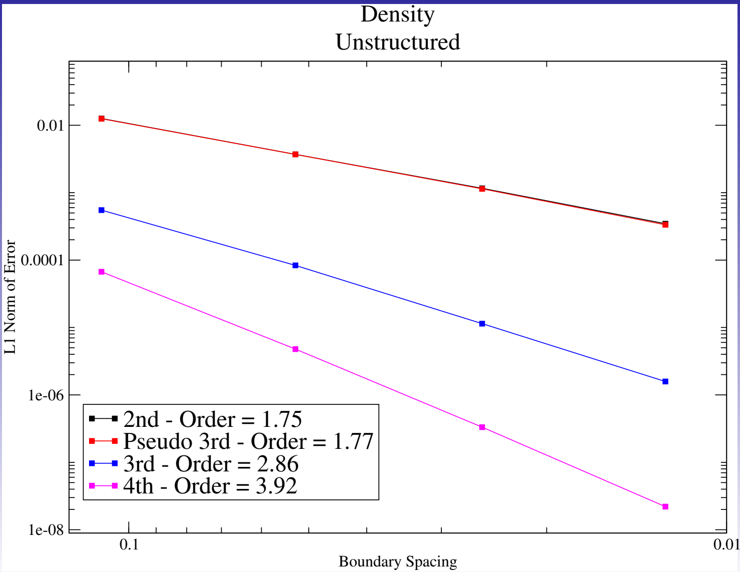
IC: $u, v = 0$ and $\rho = \frac{1}{5}$

Steady State Solutions
Supersonic Annulus, Exact Solution

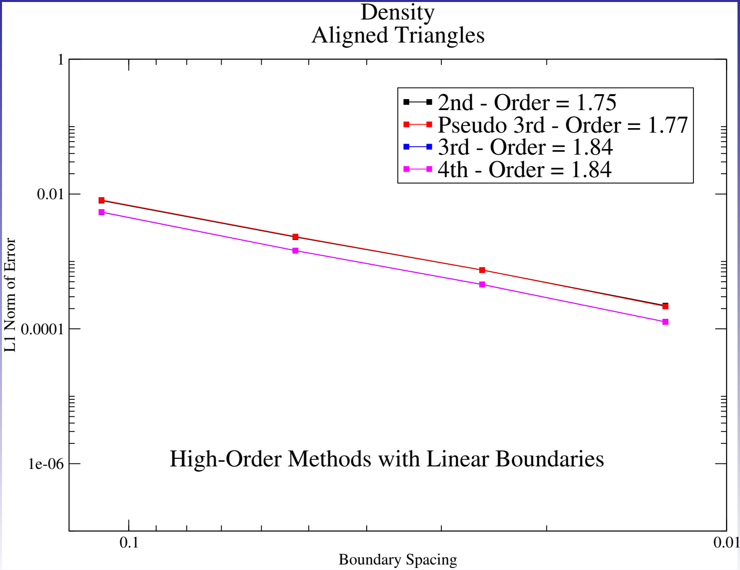


Steady State Solutions
Supersonic Annulus, continued



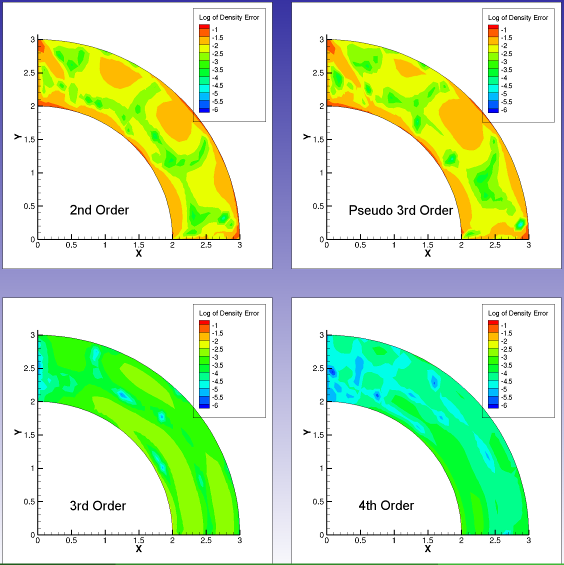

Density
Unstructured
L1 Norm of Error
0.01
0.0001
1e-06
1e-08
0.1
0.01
Boundary Spacing
2nd - Order = 1.75
Pseudo 3rd - Order = 1.77
3rd - Order = 2.86
4th - Order = 3.92

Steady State Solutions
Supersonic Annulus, continued



Steady State Solutions

Supersonic Annulus, Error in Density from Aligned Triangles (Coarsest Grid)



How Can Efficiency Be Measured?

1. Grids with Similar Error
2. Compare Time

Order	Grid Index	N_{CV}	Iterations (Total: 1^{st}, 2^{nd}, 3^{rd}, 4^{th})
2^{nd}	3	25600	340: 125,215,0,0
3^{rd}	1	1700	275: 75,50,150,0
4^{th}	0	420	300: 100,50,50,100

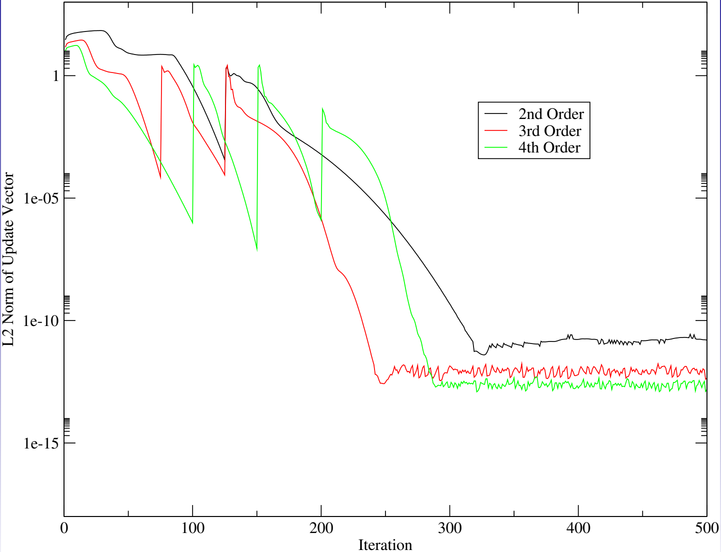
- 20 SGS Iterations (Maximum)
- CFL: 1 $\Rightarrow$ 400, 200 Iterations

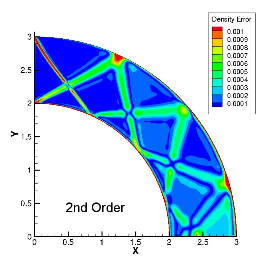
2^{nd}-order scheme	Total time =	127.220 s*, $N_{CV} = 25600$
3^{rd}-order scheme	Total time =	11.3264 s*, $N_{CV} = 1700$
4^{th}-order scheme	Total time =	4.32762 s*, $N_{CV} = 420$

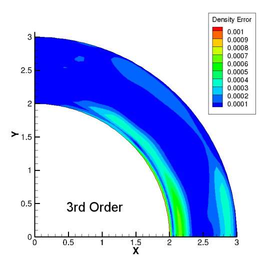
* Executed on an Intel® Core™ i5 750.

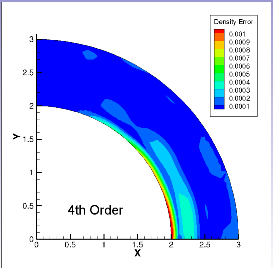
Steady State Solutions

Supersonic Annulus, Timing Results continued




Density Error
0.001
0.0009
0.0008
0.0007
0.0006
0.0005
0.0004
0.0003
0.0002
0.0001
2nd Order
Y
X


Density Error
0.001
0.0009
0.0008
0.0007
0.0006
0.0005
0.0004
0.0003
0.0002
0.0001
3rd Order
Y
X


Density Error
0.001
0.0009
0.0008
0.0007
0.0006
0.0005
0.0004
0.0003
0.0002
0.0001
4th Order
Y
X

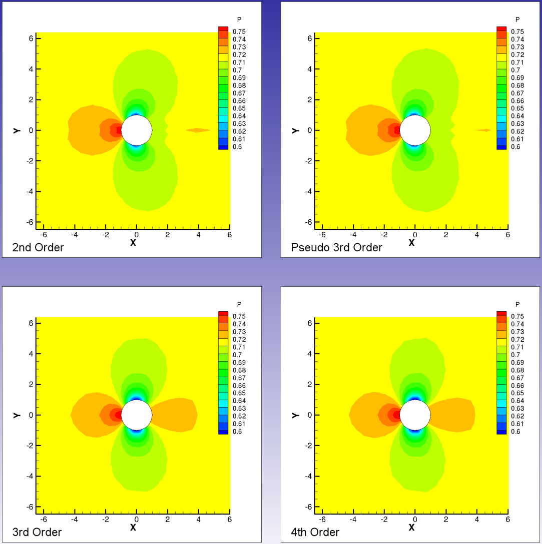
Freestream Conditions: $U_\infty = M_\infty = 0.3$

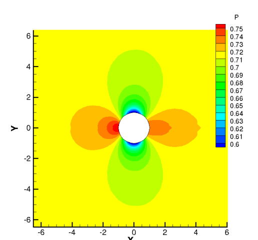
Grid Details:

Grid Index	Boundary Points	Total Points	Number of Triangles
0	48	1488	2880
1	100	4600	9000
2	200	14200	28000

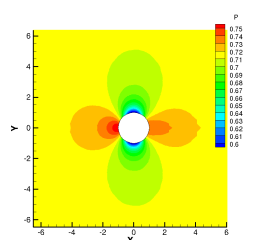
Steady State Solutions

Subsonic Circular Cylinder, Grid 0 Pressure Contours

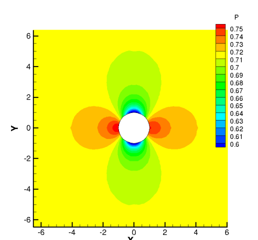




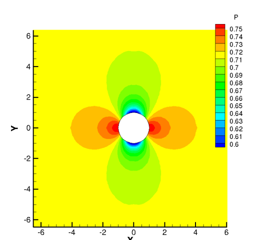
2nd Order



Pseudo 3rd Order



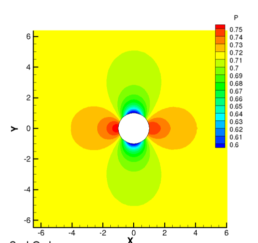
3rd Order



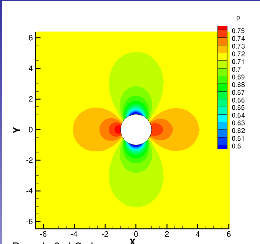
4th Order

Steady State Solutions

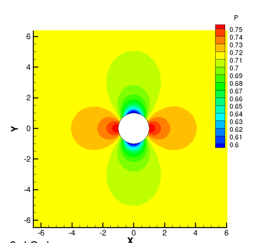
Subsonic Circular Cylinder, Grid 2 Pressure Contours



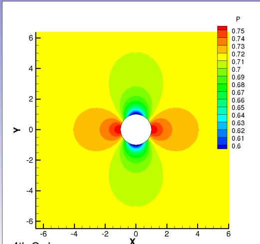
2nd Order



Pseudo 3rd Order

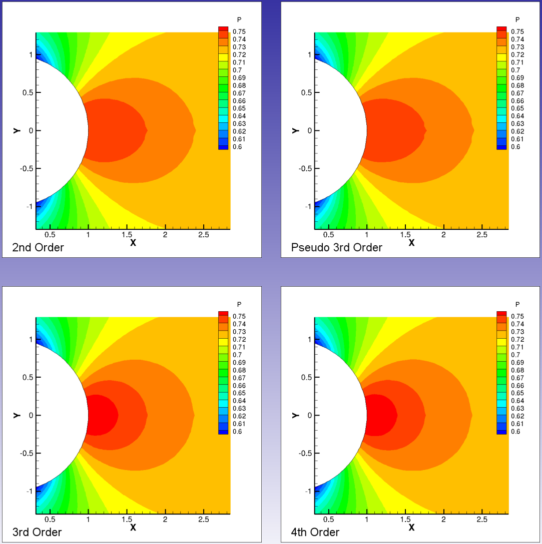


3rd Order



4th Order

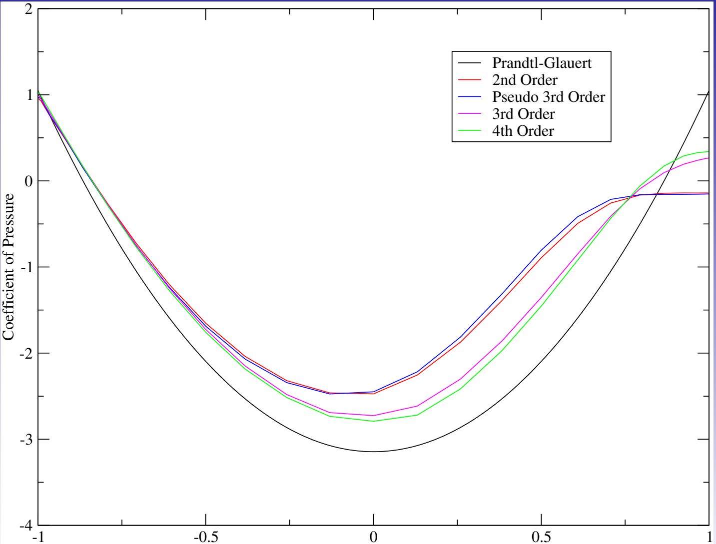
Steady State Solutions
Subsonic Circular Cylinder, Grid 2 Pressure Contours (Detail)



Potential Solution: $C_{P,i} = 1 - 4\sin^2\theta$

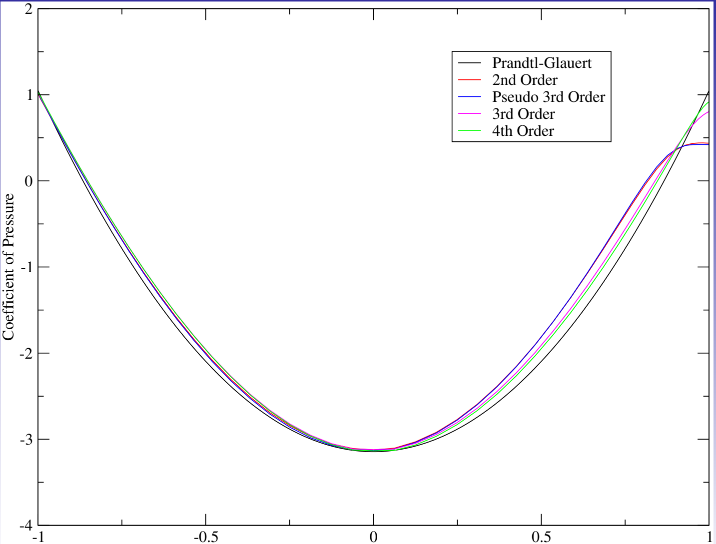
Compressible Correction - Prandtl-Glauert: $C_P = \dfrac{C_{P,i}}{\sqrt{1 - M_\infty^2}}$

Steady State Solutions
Subsonic Circular Cylinder, C_P Distribution for Grid 0



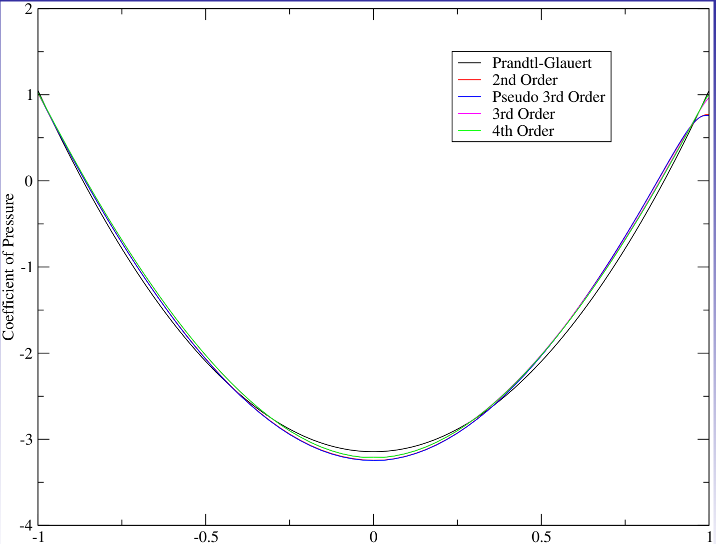
Steady State Solutions

Subsonic Circular Cylinder, C_P Distribution for Grid 1



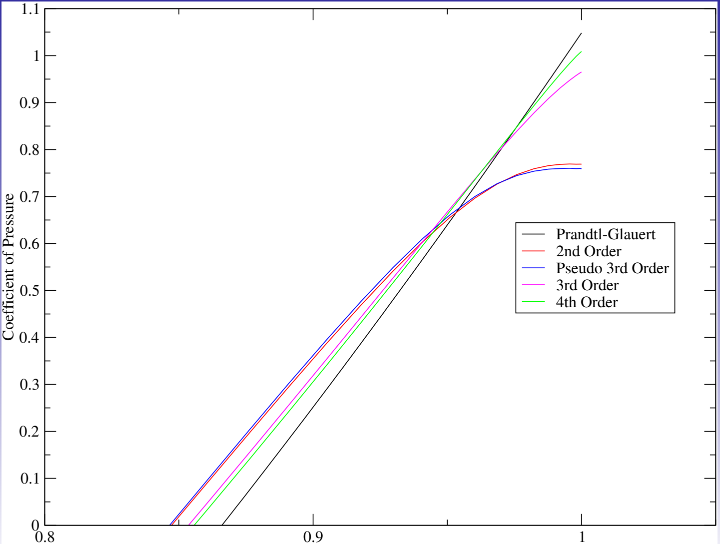
Steady State Solutions

Subsonic Circular Cylinder, C_P Distribution for Grid 2



Steady State Solutions

Subsonic Circular Cylinder, C_P Distribution for Grid 2 Detail



Second-Order Schemes Capture Front Stagnation Region Well

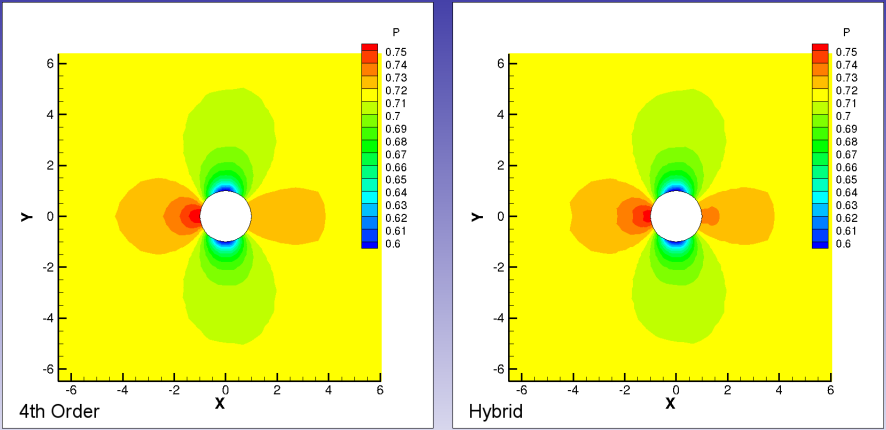
Higher Order Schemes Capture Rear Stagnation Region Better

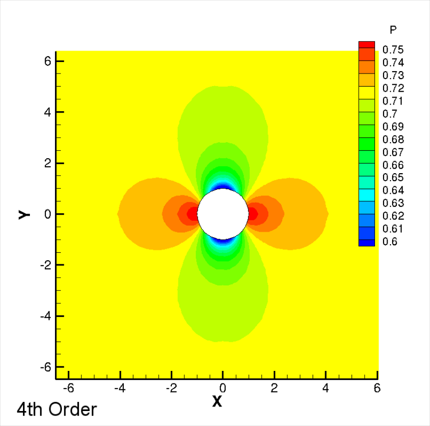
Mix Schemes:

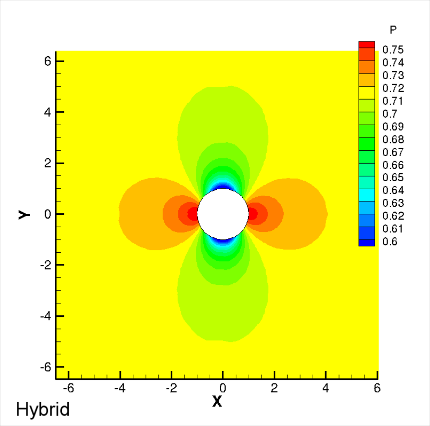
- $x < 0 \Rightarrow 2^{nd}$-Order
- $x \geq 0 \Rightarrow 4^{th}$-Order

Steady State Solutions

Subsonic Circular Cylinder, Grid 0 Pressure Contours with Hybrid Scheme

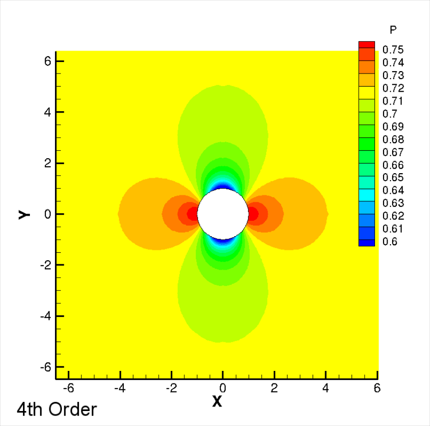


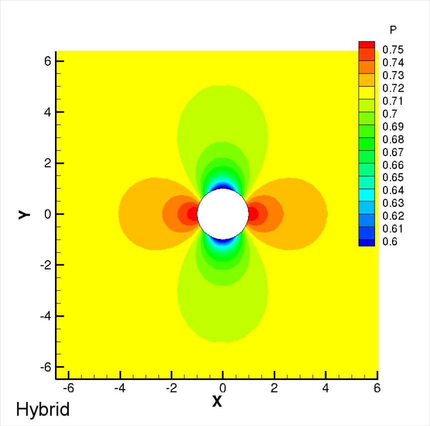

P
0.75
0.74
0.73
0.72
0.71
0.7
0.69
0.68
0.67
0.66
0.65
0.64
0.63
0.62
0.61
0.6
Y
X
-6
-4
-2
0
2
4
6
4th Order


P
0.75
0.74
0.73
0.72
0.71
0.7
0.69
0.68
0.67
0.66
0.65
0.64
0.63
0.62
0.61
0.6
Y
X
-6
-4
-2
0
2
4
6
Hybrid

Steady State Solutions

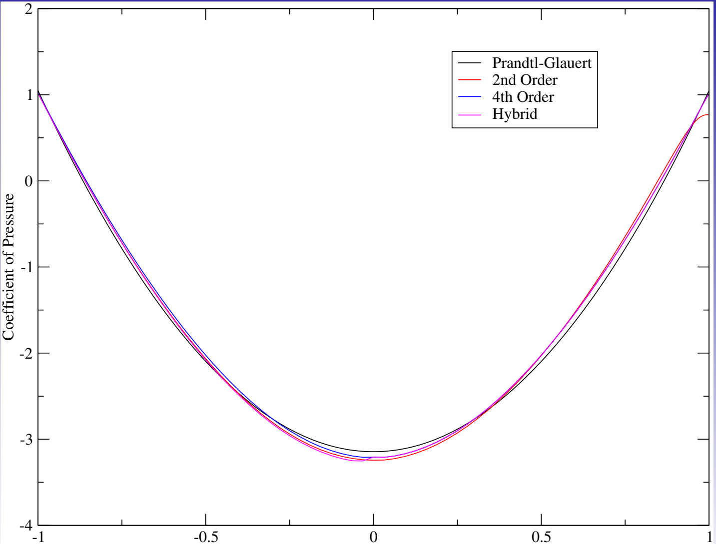
Subsonic Circular Cylinder, Grid 2 Pressure Contours with Hybrid Scheme





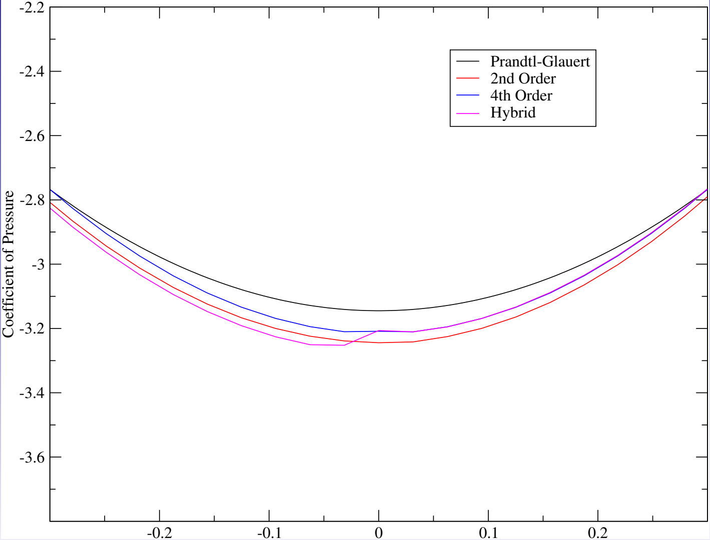
Steady State Solutions

Subsonic Circular Cylinder, C_P Distribution for Grid 2 with Hybrid Scheme



Steady State Solutions

Subsonic Circular Cylinder, C_P Distribution for Grid 2 Detail with Hybrid Scheme



Grid Details:

Grid Index	Number of nodes	Number of nodes on upper/lower surface
0	1325	125
1	3275	150
2	9188	200

- Parameterization of NACA 0012 Equation
- Distributed by Arc Length
- Newton's Method to Solve for Position

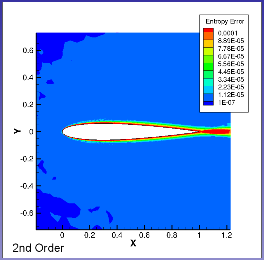
Compare Performance of Methods

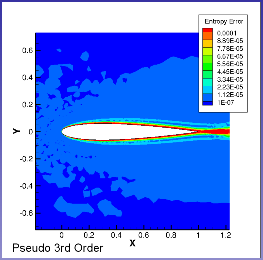
Entropy Should be Conserved

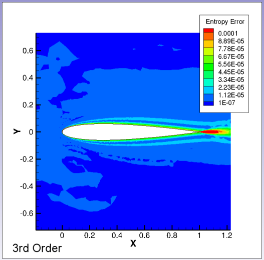
$$\frac{P}{\rho^\gamma} = Constant$$

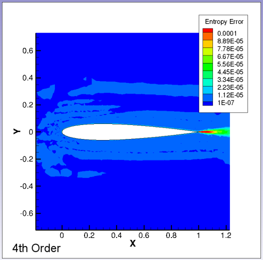
Steady State Solutions

NACA 0012 Airfoil: $M_\infty = 0.3$ and $\alpha = 0$, Visual Error



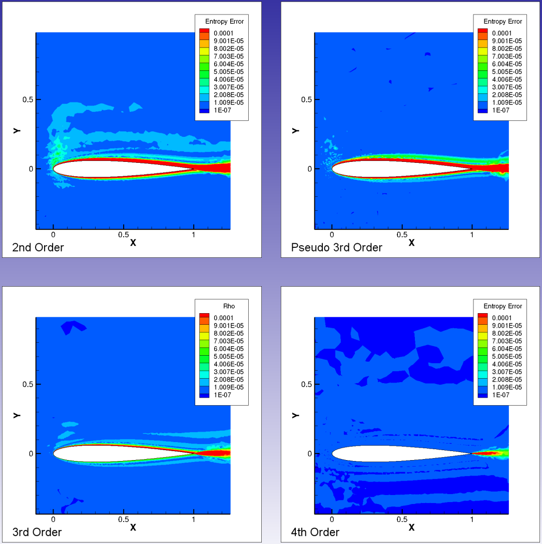






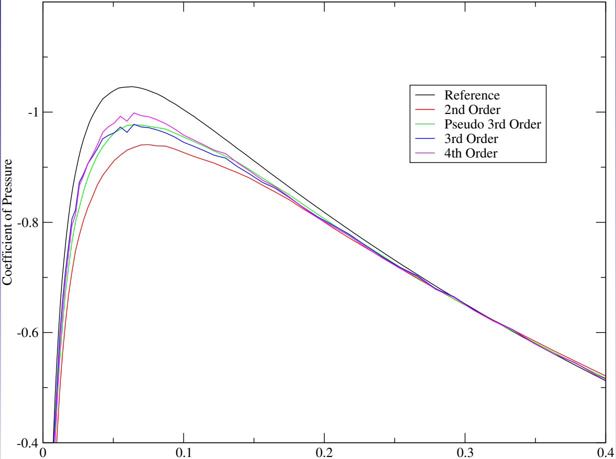
Steady State Solutions

NACA 0012 Airfoil: $M_\infty = 0.63$ and $\alpha = 2$, Visual Error



Steady State Solutions

NACA 0012 Airfoil: $M_\infty = 0.63$ and $\alpha = 2$, C_P Distribution for Grid 0



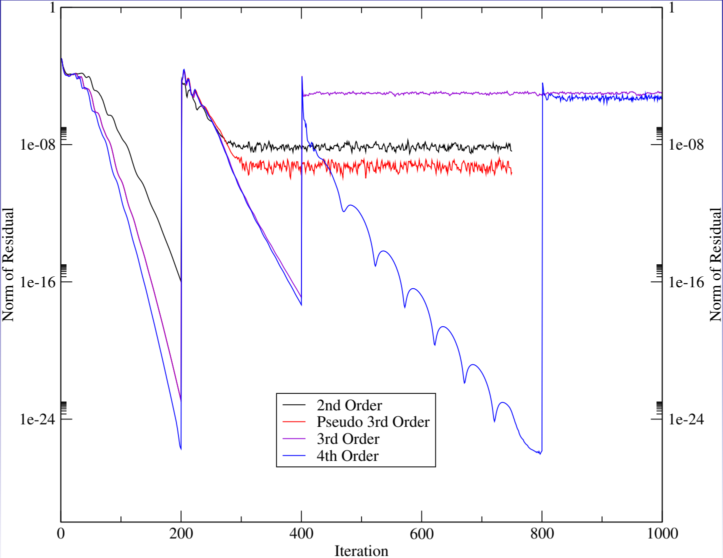
Only Consider Grid 2

Transonic $\Rightarrow$ Opportunity for Limiters

- 2^{nd}- and Pseudo 3^{rd}-Order : Venkatakrishnan Limiter
- 3^{rd}- and 4^{th}-Order : Limiter from Michalak and Ollivier-Gooch

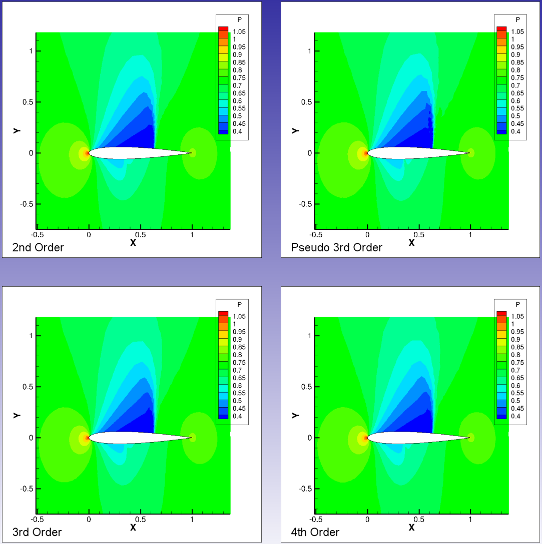
$K = 1.0$

Steady State Solutions
NACA 0012 Airfoil: $M_\infty = 0.8$ and $\alpha = 1.25$, Residual Plot



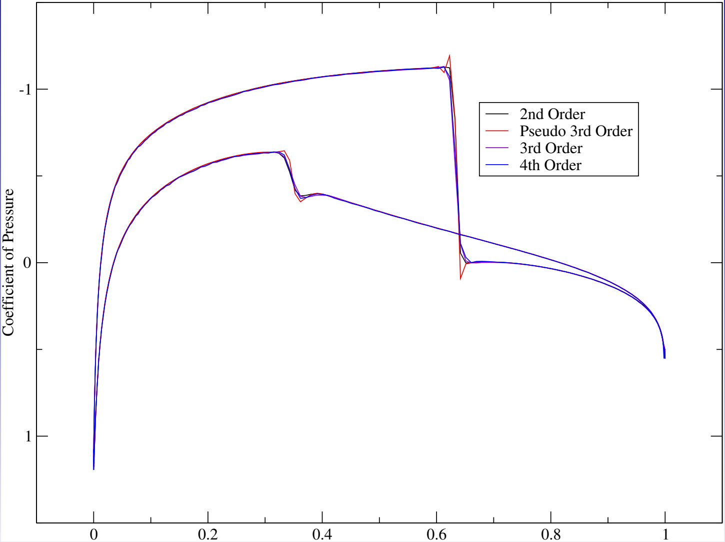
Steady State Solutions

NACA 0012 Airfoil: $M_\infty = 0.8$ and $\alpha = 1.25$, Pressure Contours



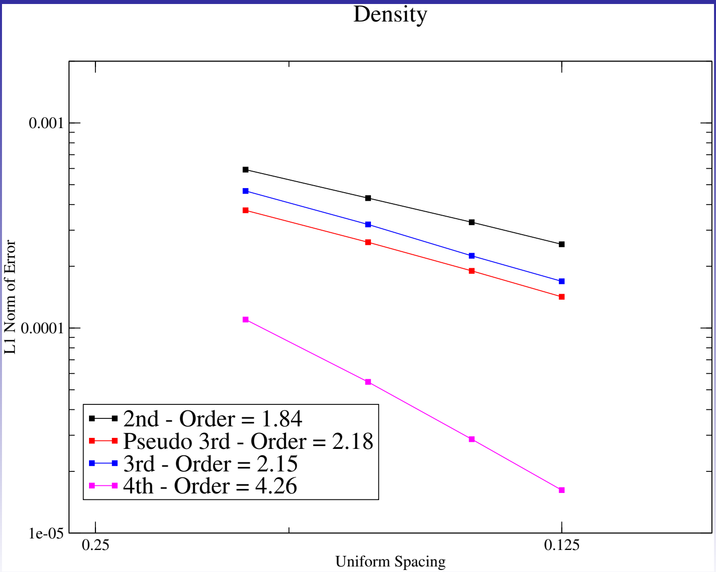
Steady State Solutions

NACA 0012 Airfoil: $M_\infty = 0.8$ and $\alpha = 1.25$, C_P Distribution



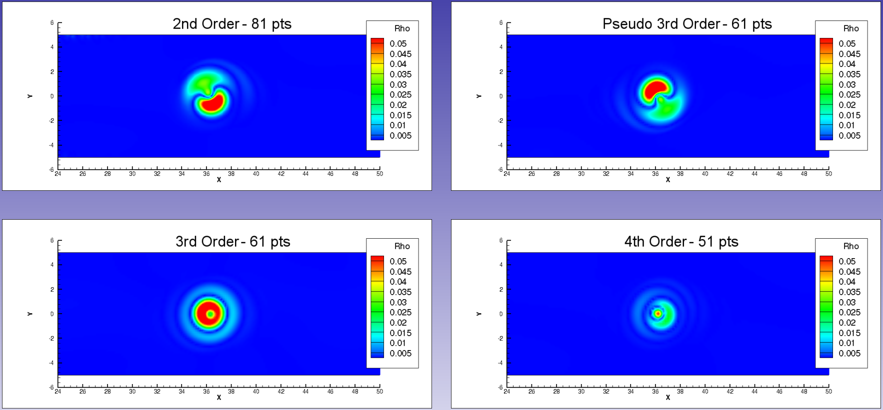
$M_\infty = 0.5$, Add an Isentropic Vortex
Grids Span $[0, -5] \times [150, 5]$, $\Delta t = 0.0125$, 5000 Iterations Applied

Grid Index	Points in y	Points in x	Total Points	Δx
0	51	751	38301	0.2
1	61	901	54961	0.1667
2	71	1051	74621	0.1429
3	81	1201	97281	0.125
*	41	601	24641	0.25

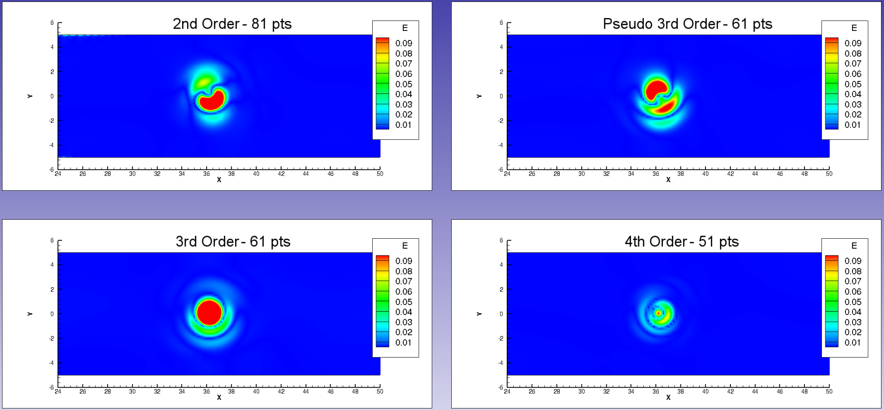

Density
0.001
0.0001
1e-05
L1 Norm of Error
0.25
0.125
Uniform Spacing
2nd - Order = 1.84
Pseudo 3rd - Order = 2.18
3rd - Order = 2.15
4th - Order = 4.26

Unsteady Solutions

Vortex Convection, Density Error Contours



Unsteady Solutions
Vortex Convection, Total Energy Error Contours



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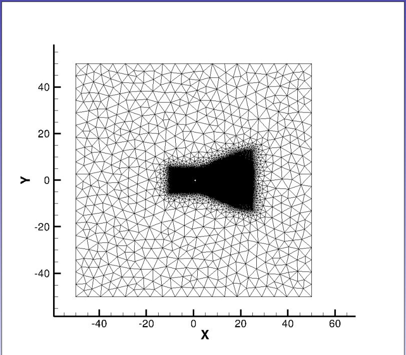
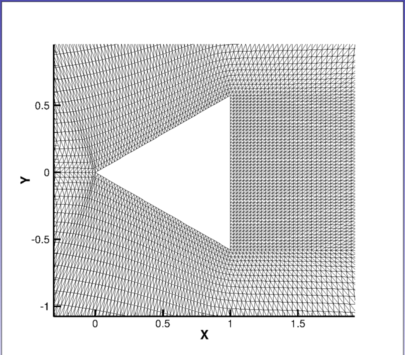
THE UNIVERSITY of TENNESSEE at CHATTANOOGA

Order, Grid	ρ	ρu	ρv	E
2^{nd}, 81	2.56e-04	6.07e-04	4.82e-04	6.61e-04
pseudo 3^{rd}, 61	2.62e-04	5.45e-04	4.73e-04	6.84e-04
3^{rd}, 61	3.20e-04	5.44e-04	4.70e-04	8.51e-04
4^{th}, 41	2.05e-04	3.45e-04	3.32e-04	4.70e-04

2^{nd}-order scheme, 81	Total time =	15.1 hrs*
Pseudo 3^{rd}-order scheme, 61	Total time =	8.7 hrs*
3^{rd}-order scheme, 61	Total time =	13.5 hrs*
4^{th}-order scheme, 41	Total time =	7.7 hrs*

* Executed on an Intel® Core™ i5 750.

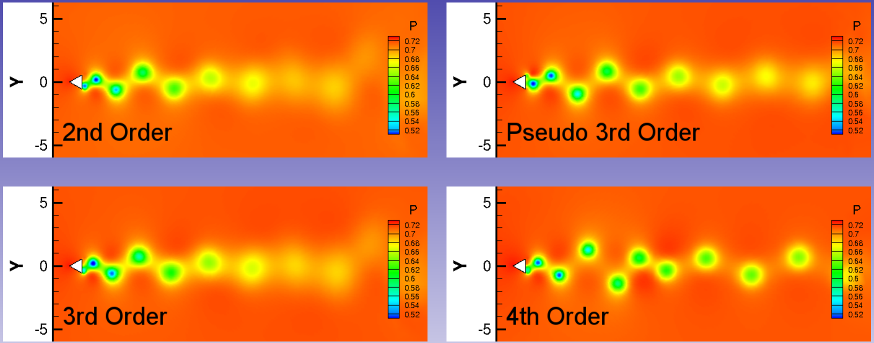

40
20
0
-20
-40
Y
-40
-20
0
20
40
60
X


0.5
0
-0.5
-1
Y
0
0.5
1
1.5
X

- Grid: 41217 Points, 82211 Triangles
- $M_\infty = 0.2$
- 800 1^{st}-Order Iterations (Steady)
- Restart with Appropriate Order (Unsteady, $\Delta t = 0.05$)
- Run Until Iteration 20000

Unsteady Solutions

Vortex Shedding Over a Wedge, Pressure Contours



2^{nd}-order scheme	Total time =	3.5 days*
Pseudo 3^{rd}-order scheme	Total time =	4.3 days*
3^{rd}-order scheme	Total time =	8.5 days*
4^{th}-order scheme	Total time =	15.9 days*

* Executed on an Intel® Xeon® X7560.

	2^{nd}	pseudo 3^{rd}	3^{rd}	4^{th}
Newton Iterations	20	26	30	40

Computational Expense

Average time per node over iterations (time steps and Newton):

- 2^{nd}-Order : $19\mu s$
- Pseudo 3^{rd}-Order : $18\mu s$
- 3^{rd}-Order : $31\mu s$
- 4^{th}-Order : $43\mu s$

- Solver with High-Order Spatial Accuracy
- Accuracy Demonstrated with MMS
- Accuracy Demonstrated with Grid Convergence
- Proper Curved Boundaries
- Slope Limiters
- Method Works for Unsteady Problems

- Add Viscous Terms
- Parallelization
- Extend to Tenasi (Some of this is done.)

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Committee - Drs. Daniel Hyams and Steve Karman

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